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# Seiberg duality in Chern–Simons theory

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**Abstract**

We argue that $N = 2$ supersymmetric Chern–Simons theories exhibit a strong–weak coupling Seiberg-type duality. We also discuss supersymmetry breaking in these theories.


## 1. Introduction

Three-dimensional Chern–Simons (CS) gauge theories with $N = 2$ supersymmetry (i.e. four real supercharges) coupled to "matter" chiral superfields give rise to a large class of quantum field theories with non-trivial infrared dynamics (see e.g. [1,2] and references therein). These theories are characterized by a gauge group $G$, Chern–Simons level $k$, and matter representation $R$. They are classically conformal, since the level $k$, which plays the role of a coupling constant, is dimensionless. The conformal symmetry extends to the quantum theory since $k$ does not run along Renormalization Group (RG) trajectories. Indeed, for non-Abelian gauge groups $k$ is quantized, and thus cannot run.

One can also add superpotential interactions among the matter superfields. In general, these break the conformal symmetry and generate non-trivial RG flows. In some cases they modify the infrared behavior.

Determining the quantum dynamics of these theories is an interesting problem, which in many ways is reminiscent of the analogous problem in four-dimensional Yang–Mills theories with $N = 1$ supersymmetry. However, while in four dimensions much progress has been

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made by using the NSVZ $\beta$-function [3], Seiberg duality [4], $a$-maximization [5–8], etc., in three-dimensional CS theory the understanding is more rudimentary. For large $k$ one can use perturbation theory in $1/k$, but in general the problem is unsolved.

As mentioned above, one of the important tools in analyzing the infrared dynamics of four-dimensional $N=1$ supersymmetric gauge theories is Seiberg duality, which in many cases maps a strongly coupled gauge theory to a weakly coupled or IR free one. In this note we will propose an analog of Seiberg duality for three-dimensional $N=2$ supersymmetric Chern–Simons theories. While this duality is a field theory phenomenon, we will phrase the discussion in terms of brane constructions that reduce to the relevant field theories at low energies. These constructions capture efficiently both classical and quantum aspects of CS dynamics.

The rest of this note is organized as follows. We start by describing the brane configuration we will be interested in and its low energy CS description. We then use the results of [9] to construct the Seiberg dual configuration and analyze its low energy limit. We discuss the relation between the two CS theories, and propose that they are equivalent. We also describe supersymmetry breaking vacua that generalize the ISS [10] construction to CS theory. Unlike their four-dimensional analogs, these vacua appear to be stable.

## 2. Electric theory

We will study brane configurations in type IIB string theory that involve two types of NS5-branes, which we will denote by NS and NS′, as well as D3-branes and D5-branes. The different branes are oriented as follows in $\mathbb{R}^{9,1}$:

$$\begin{aligned} &\mathrm{NS}: && (012345),\\ &\mathrm{NS}': && (012389),\\ &\mathrm{D3}: && (0126),\\ &\mathrm{D5}: && (012789). \end{aligned} \tag{2.1}$$

These are precisely the branes that are used in the Hanany–Witten construction [11] of three-dimensional gauge theories (see [12] for a review). It is not difficult to check that a configuration which includes all the branes in (2.1) preserves $N=2$ supersymmetry in the three dimensions common to all the branes, (012).

When an NS′-brane intersects $k$ D5-branes in the (37)-plane, the two types of branes can locally combine into a $(1,k)$ fivebrane (see Fig. 1), which is oriented at an angle $\theta$ to the NS′-brane, with $\tan\theta = g_s k$ [13]. The resulting brane configuration preserves supersymmetry for all values of the length of the $(1,k)$ fivebrane segment. When the length of this segment goes to infinity, the NS′-brane and D5-branes are replaced by the $(1,k)$ fivebrane everywhere; the supersymmetry is not affected.

The brane configuration we consider is depicted in Fig. 2(a), where we use the notation:

$$v = x^4 + ix^5, \qquad w = x^8 + ix^9, \qquad y = x^6. \tag{2.2}$$

The corresponding low energy theory is a $U(N_c)$ gauge theory with $N_f + k$ flavors of chiral superfields $Q^i$, $\tilde{Q}_i$ in the fundamental representation of the gauge group [12].

In order to study the dynamics of interest, we move $k$ of the D5-branes towards the NS′-brane, and when the two intersect, deform the configuration as in Fig. 1, such that the NS′-brane and $k$ D5-branes are replaced by a $(1,k)$ fivebrane. The resulting brane configuration appears

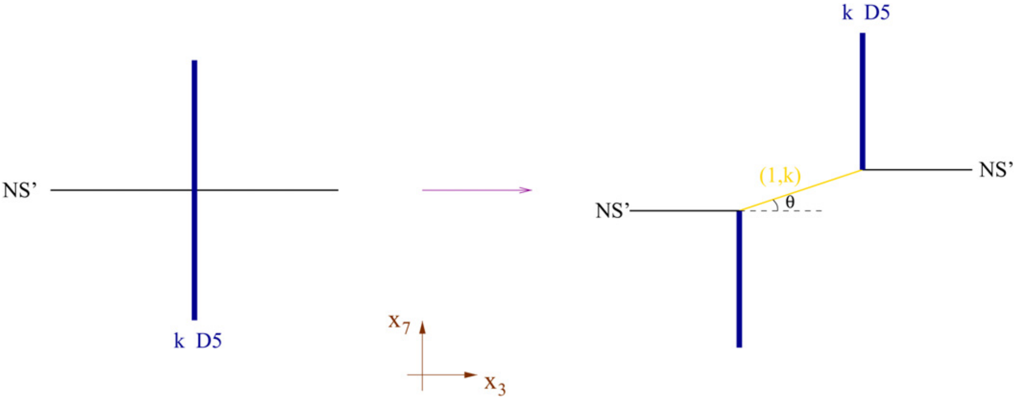


Fig. 1. Fivebrane recombination.

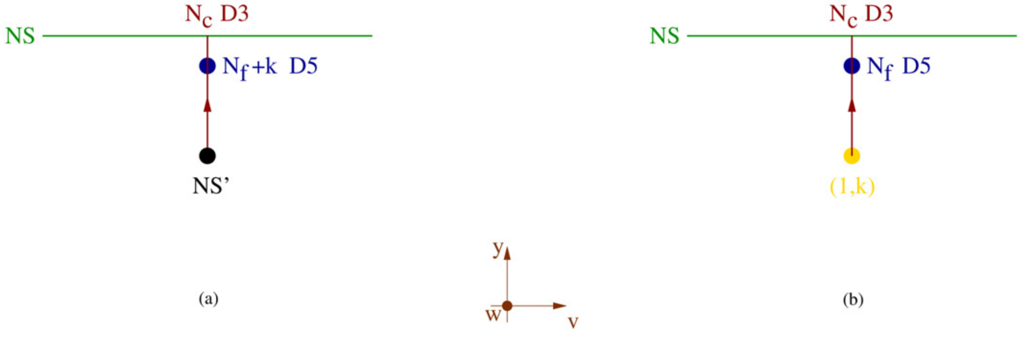


Fig. 2. Electric brane configuration.

in Fig. 2(b). The deformation that takes Fig. 2(a) to Fig. 2(b) corresponds in the field theory to giving real masses of the same sign to the $k$ flavors of fundamentals $Q$ and $\tilde{Q}$ that were singled out in the construction [14]. The limit in Fig. 2(b) corresponds to sending these masses to infinity.

The low energy limit of the brane configuration of Fig. 2(b) is described by a level $k$ $U(N_c)$ CS theory [15], coupled to $N_f$ fundamentals $Q^i$, $\tilde{Q}_i$, $i = 1, 2, \ldots, N_f$. It preserves $N = 2$ superconformal symmetry. In the remainder of this section we briefly comment on some of its properties.

The global symmetry of the gauge theory is $SU(N_f) \times SU(N_f) \times U(1)_a \times U(1)_R$. The first three factors can be seen in the brane picture by starting with the configuration of Fig. 2(b), moving all $N_f$ D5-branes to the $(1,k)$-brane, and performing separate $U(N_f)$ transformations on the portions of the D5-branes with $x^7 > 0$ and $x^7 < 0$ [16]. The $U(1)_R$ is a subgroup of the $(9+1)$-dimensional Lorentz group preserved by the brane configuration.

One can also use the brane picture to identify some of the perturbations and moduli of the low energy field theory [12]. In particular, moving the D5-branes in the $v$ direction corresponds to turning on complex masses for $Q^i$, $\tilde{Q}_i$ via a superpotential of the form $W = m_i \tilde{Q}_i Q^i$. Moving them in the $x^3$ direction corresponds to giving real masses with opposite signs to $Q$, $\tilde{Q}$.

The moduli of the CS theory can be seen geometrically exactly as in the four-dimensional $N = 1$ case, by separating the $N_f$ D5-branes in Fig. 2(b) in $x^6$, and allowing the D3-branes to break on them (see e.g. Fig. 25 in [12]). One finds, as there, that the dimension of the moduli

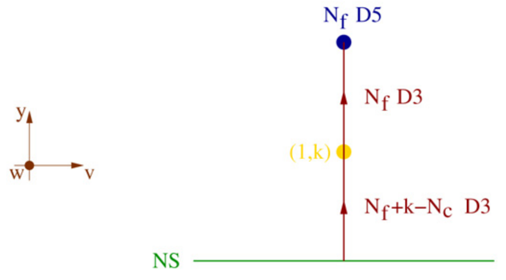


Fig. 3. Magnetic brane configuration.

space is given by

$$\dim \mathrm{M} = \begin{cases} N_f^2, & N_f < N_c, \\ 2N_cN_f - N_c^2, & N_f \geqslant N_c. \end{cases} \tag{2.3}$$

The classical analysis above receives quantum corrections due to the following effect. It was shown in [14,17] that the number of D3-branes that can stretch between an NS-brane and a $(1,k)$-brane without breaking supersymmetry is bounded from above by $k$. This is a consequence of the "s-rule" of [11], and is related to the fact that such D3-branes are necessarily on top of each other. At first sight it seems that this implies that in the configuration of Fig. 2(b) there is no supersymmetric vacuum unless $N_c \leqslant k$, but the actual bound is less restrictive.

The reason is that one can think of $N_f$ out of the $N_c$ D3-branes[1] in Fig. 2b as stretching from the NS-brane to the D5-branes and then from the D5-branes to the $(1,k)$-brane, so the net number of D3-branes that enters the bound of [14,17] is $N_c - N_f$. Hence, we conclude that the CS theory corresponding to Fig. 2(b) has a supersymmetric vacuum for

$$N_f + k - N_c \geqslant 0. \tag{2.4}$$

When (2.4) is satisfied, the quantum moduli space has the dimension (2.3). Note that the constraint (2.4) allows $N_f$ to be either smaller or larger than $N_c$. Note also that although we presented the derivation of (2.4) in brane terms, it is a property of the CS theory [14,17,18].

## 3. Magnetic theory and duality

In order to construct the dual theory, we follow [9] and exchange the NS and $(1,k)$ fivebranes. A convenient way to do this is to go back to the configuration of Fig. 2(a), move all $N_f + k$ D5-branes to the other side of the NS-brane, creating $N_f + k$ D3-branes in the process [11], and then move the NS′-brane through the NS-brane. Finally, we need to recombine the $k$ D5-branes with the NS′-brane into a $(1,k)$-brane, as in the transition from Fig. 2(a) to Fig. 2(b). The resulting brane configuration is depicted in Fig. 3.

The low energy effective field theory can be read off Fig. 3 as in [12,15]. It includes a level $k$ $U(N_f + k - N_c)$ CS gauge field coupled to $N_f$ fundamentals $q_i$, $\tilde{q}^i$, as well as an $N_f \times N_f$ matrix of singlets $M^i_j$, which couple to the fundamentals via the superpotential

$$W = M^i_j q_i \tilde{q}^j. \tag{3.1}$$

[1] It is enough to consider the case $N_c \geqslant N_f$.

It is thus natural to propose that this magnetic CS theory is dual to the electric one discussed in the previous section, with the usual identification

$$M^i_j = Q^i \tilde{Q}_j. \tag{3.2}$$

Note that the constraint (2.4), which is necessary for having a supersymmetric vacuum, is in the magnetic theory just the requirement that the rank of the magnetic gauge group is non-negative. This is reminiscent of what happens in four-dimensional $N = 1$ supersymmetric QCD, where the analogous constraint is $N_f - N_c \geqslant 0$.

Note also that unlike the four-dimensional case, it is important here that the duality involves $U(N_c)$ and $U(N_f + k - N_c)$ and not the corresponding $SU$ groups. Indeed, the $U(1)$ factor is interacting in this case, and it is easy to see that if it was not gauged, the duality could not be correct.

As a check of the duality, we may ask whether the magnetic CS theory reproduces the moduli space of vacua of the electric theory, whose dimension is given by (2.3). Naively, it looks like the moduli space of the brane configuration of Fig. 3 is $N_f^2$-dimensional, with the counting being the same as in Fig. 29 in [12].

For $N_f \leqslant N_c$ this answer is correct, but for $N_f > N_c$ it is important to take into account the constraint on the number of D3-branes stretched between the NS and $(1, k)$ fivebranes, which played a role in the derivation of (2.4). Indeed, in this case, at a generic point in the $N_f^2$-dimensional classical moduli space of Fig. 29 of [12], we have in Fig. 3 $N_f + k - N_c > k$ D3-branes stretched between the fivebranes, which as mentioned before leads to a non-supersymmetric state. To preserve supersymmetry, we must keep $N_f - N_c$ of the flavor D3-branes at the origin. It is easy to check that taking this into account leads to precise agreement with the electric result (2.3).

We see that while the constraint (2.4) arises from quantum effects in the electric theory and is a classical property of the magnetic one, the opposite happens in the analysis of the moduli space: the dimension (2.3) is obtained classically in the electric theory, and requires quantum effects in the magnetic one.

Another class of deformations involves giving masses to some of the flavors. Turning on the superpotential $W = m_1 \tilde{Q}_1 Q^1$ in the electric theory, corresponds in Fig. 2(b) to separating one of the $N_f$ D5-branes in the $v$ direction from the D3-branes. Integrating out $Q^1$, $\tilde{Q}_1$ amounts to sending this separation to infinity. In the magnetic configuration, of Fig. 3, this deformation requires the D3-brane connected to that D5-brane to combine with one of the $N_f + k - N_c$ D3-branes stretched between the $(1, k)$ and NS fivebranes, thus reducing the rank of the gauge group by 1. This leads, as in the four-dimensional case [4,9], to a dual pair with $N_f \to N_f - 1$, with all other parameters remaining the same.

Giving equal and opposite sign real masses to $Q^1$, $\tilde{Q}_1$ corresponds in Fig. 2(b) to moving the corresponding D5-brane away from the D3-branes in the $x^3$ direction. There are now two types of supersymmetric vacua. In one, the electric gauge group remains unbroken, i.e. the D3-branes continue to stretch between the NS and $(1, k)$ fivebranes. Sending the displaced D5-brane to infinity amounts to reducing $N_f$ by one unit while keeping all the other parameters fixed, as before.

A second vacuum is obtained by allowing one of the $N_c$ D3-branes to break on the displaced D5-brane, such that as it moves in $x^3$, half of the D3-brane stretches between the NS and D5 branes, while the other half stretches between the D5 and $(1, k)$ branes. As the D5-brane is sent to infinity, one finds a vacuum of the original kind, with both $N_f$ and $N_c$ reduced by one unit.

In the magnetic brane configuration of Fig. 3, one finds the same vacua. Displacing one of the D5-branes in $x^3$, one finds again two types of supersymmetric configurations. In one, the D5-brane drags with it the D3-brane attached to it, reducing $N_f$ by one but not changing the rank of the magnetic gauge group. This gives rise to the magnetic dual of the second kind of electric vacuum discussed above.

The dual of the first kind of electric vacuum is obtained by reconnecting the D3-brane attached to the mobile D5-brane to one of the color D3-branes, and then moving the D5-brane in $x^3$. This gives rise to a vacuum in which both the number of flavor and that of colors in the magnetic theory are reduced by one unit, in agreement with expectations.

Finally, giving same sign real masses to $Q^1$, $\tilde{Q}_1$ corresponds in the electric brane configuration of Fig. 2(b) to moving a D5-brane in $x^6$ towards the $(1,k)$ brane, and using the process of Fig. 1 to turn it into a $(1,k+1)$ brane. This leads to the same type of theory, with $N_f \to N_f - 1$, $k \to k+1$.

Similarly, in the magnetic configuration of Fig. 3, we need to send a D5-brane towards the $(1,k)$ fivebrane and make the transition of Fig. 1. This again corresponds to taking $N_f \to N_f - 1$ and $k \to k+1$. Note that the rank of the magnetic gauge group does not change in the process, in agreement with the duality.

To summarize, we see that the duality proposed above is consistent with the structure of moduli space and deformations. This duality is a strong–weak coupling one in the following sense. Consider first the electric theory. The interactions between the chiral superfields $Q^i$, $\tilde{Q}_i$ are due to the CS coupling $k$. Thus, if we keep $N_c$, $N_f$ fixed and send $k \to \infty$, the electric theory becomes more and more weakly coupled. Note that in this limit the quantum constraint (2.4) is automatically satisfied, as one would expect. On the other hand, for $k$ of order $N_c$ the electric theory is strongly coupled.

In the magnetic theory, we have two kinds of interactions. One is due to the $U(N_f + k - N_c)$ CS gauge field; the other due to the cubic superpotential (3.1). Let us first ignore the superpotential and focus on the gauge interaction. In the regime where the electric CS interaction is weakly coupled, the rank of the magnetic gauge group $\bar{N}_c = N_f + k - N_c$ is of order $k$. Thus, it is strongly coupled. To make the magnetic CS theory weakly coupled, one needs to consider the regime $k \gg \bar{N}_c$. This can be achieved, for example, by keeping $N_f$ and $\bar{N}_c$ fixed and sending $k \to \infty$. In this limit $N_c \simeq k$ so the electric CS theory is strongly coupled.

Even when the magnetic CS gauge interaction is weak, the theory still contains a cubic superpotential, (3.1), which is a relevant perturbation that grows in the infrared. We are not going to say much about it here, except to note that:

(1) One can go to the regime $k \gg \bar{N}_c \gg 1$ with $N_f$ fixed (say), in which the Wess–Zumino model with superpotential (3.1) is a large $N$ vector model, which can presumably be solved using standard large $N$ techniques. In this sense it is weakly coupled, with the small coupling being $1/\bar{N}_c$.

(2) One can put the electric and magnetic theories on the same footing by adding to the electric theory a quartic superpotential[2]

$$W = \lambda(\tilde{Q}Q)^2. \tag{3.3}$$

[2] Such superpotentials in four-dimensional $N = 1$ SQCD and their brane realizations have been recently studied in [19,20].

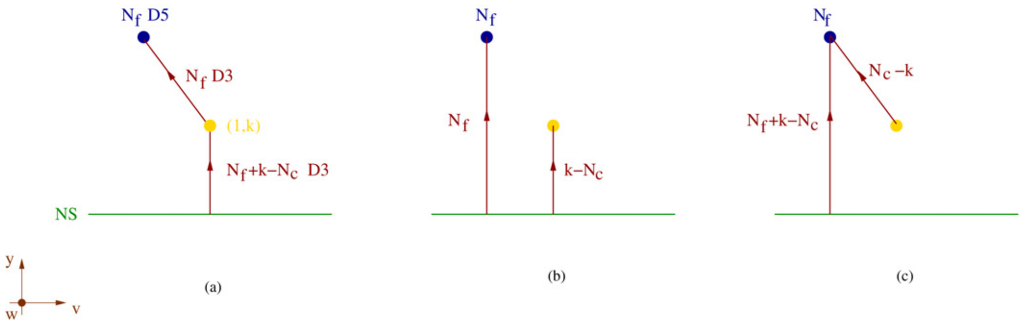


Fig. 4. Supersymmetric and non-supersymmetric vacua of the magnetic theory for non-zero mass.

Under the duality we proposed here, this corresponds to adding $W = \lambda M^2$ to (3.1). Integrating out $M$ leads to a quartic superpotential for the magnetic quarks very similar to (3.3). The resulting infrared theory preserves $N = 3$ superconformal symmetry, and can be made arbitrarily weakly coupled by tuning $k$, $N_f$ and $N_c$.

## 4. Supersymmetry breaking

In four dimensions, it was shown in [10] that the magnetic dual of $N = 1$ supersymmetric QCD with a small mass deformation[3],

$$W = m\tilde{Q}_i Q^i, \tag{4.1}$$

has a metastable supersymmetry breaking vacuum. It is interesting to ask what happens in our case. Consider first the electric theory. As discussed above, turning on the mass term (4.1) corresponds in the brane construction of Fig. 2(b) to displacing all $N_f$ D5-branes in the $v$ direction, by the same amount. The resulting configuration has $N_c$ D3-branes stretched between the NS and $(1, k)$ fivebranes with no D5-branes to screen them, so the s-rule implies that it is only supersymmetric when

$$N_c \leqslant k, \tag{4.2}$$

a stronger constraint than (2.4). In particular, for $N_c > k$ supersymmetry is spontaneously broken.

To connect to the discussion of [10] consider the magnetic theory of Fig. 3. The mass deformation (4.1) corresponds to adding to the magnetic superpotential (3.1) the term $\delta W = mM$. In the brane construction, this corresponds again to moving the $N_f$ D5-branes in the $v$ direction. This gives rise to the configuration of Fig. 4(a). This configuration is non-supersymmetric; its fate depends on whether the inequality (4.2) is satisfied. If it is, there are more color threebranes than flavor ones, so they reconnect and lead to the configuration of Fig. 4(b), which is the supersymmetric vacuum dual to that discussed above in the electric theory.

For $N_c > k$ there are not enough color branes to combine with all the flavor ones, and the ground state of the system corresponds to the configuration of Fig. 4c. This configuration is non-

3 We restrict here to the case of equal masses for all the flavors. In four dimensions, new effects appear when some of the masses are zero [21]; it would be interesting to investigate the analogous problem in the CS case.

supersymmetric. It is clearly a direct analog of the four-dimensional configurations of [22–25]. The difference is that while there these configurations were metastable, and there was a supersymmetric vacuum elsewhere in field space, here we expect this supersymmetry breaking vacuum to be stable.

A quick way to see this is that in [22–25] the electric brane configuration had supersymmetric vacua, so the magnetic one must have them as well, by duality, whereas here the electric theory breaks supersymmetry (for $N_c > k$). Also, in the four-dimensional brane construction it is known that certain quantum effects, which are needed for constructing the supersymmetric vacuum in the magnetic theory, are difficult to see in the brane construction [12], whereas in the three-dimensional brane constructions discussed here the quantum effects are expected to be visible in the brane description.

Coming back to Fig. 4(c), like in the four-dimensional brane configurations of [22–25], the $N_c - k$ D3-branes stretched between the D5-branes and the $(1, k)$ fivebrane give rise naively to (pseudo-)moduli, corresponding to their motion in the $w$ plane, in which both kinds of fivebranes are extended. In the brane description it is clear that these moduli are absent due to the attraction of the D3-branes to the NS-brane [25]. Thus, the supersymmetry breaking vacuum of Fig. 4(c) is stable.

In four dimensions, the analog of the brane attraction in weakly coupled magnetic SQCD is the one-loop potential for the pseudo-moduli computed in [10]. We expect something similar to happen in the three-dimensional case, but have not computed the potential for the pseudo-moduli directly.

## 5. Discussion

In this note we proposed that $N = 2$ supersymmetric level $k$ $U(N_c)$ Chern–Simons theory with $N_f$ fundamental chiral superfields $Q^i$, $\tilde{Q}_i$ has a dual description, in which the gauge group is replaced by $U(N_f + k - N_c)$, and the chiral superfields are fundamentals $q_i$, $\tilde{q}^i$ as well as singlets $M^i_j$, coupled via the superpotential (3.1). This duality exchanges regions with strong and weak CS coupling; in this sense, it is a strong–weak coupling duality.

We presented the duality in terms of brane configurations in type IIB string theory, but it is a property of CS theory. The brane description provides a convenient geometric language in terms of which one can study the moduli spaces and deformations, both classically and quantum-mechanically, but the whole discussion could be repeated in field theory language.

A generalization of Seiberg duality to three-dimensional $N = 2$ supersymmetric gauge theory was previously proposed in [26,27] (and further discussed from the brane perspective in [12]). In these works the kinetic term of the gauge field had the standard Yang–Mills form, and the CS term was absent. This leads to some differences with our analysis.

First, because the gauge coupling is dimensionful in three dimensions, in [26,27] both the electric and the magnetic theories are strongly coupled in the infrared. Thus, the dualities of [26,27] are strong–strong coupling ones. Second, since the mass of the gauge field provided by the CS term is absent, there are additional chiral superfields, associated with the vector superfield along the Coulomb branch of the theory, which are difficult to define microscopically.

At the same time, the dualities of [26,27] are closely related to the one described here. This is clear from the brane description we used. Indeed, before performing the deformation of Fig. 1 for $k$ D5-branes on the electric and magnetic sides, the infrared limits of the electric and magnetic brane configurations are precisely those of [27]. In other words, assuming the dualities of [26,27], our results can be derived by turning on real masses for some of the flavors.

From this point of view, our main point is that turning on these real masses, eliminates both of the problematic features of the dualities of [26,27]. By giving a mass to the gauge field, it eliminates the Coulomb branch and the associated degrees of freedom, and by replacing the Yang–Mills kinetic term with the CS one, it opens the possibility of having a strong–weak coupling duality.

There are many questions along the lines of our discussion that require further work. For example, in four dimensions, $N = 1$ supersymmetric $SU(N_c)$ SYM theory with an adjoint chiral superfield $X$ and $N_f$ fundamentals $Q^i$, $\tilde{Q}_i$, exhibits a generalization of Seiberg duality when we turn on a polynomial superpotential for $X$, $W = \mathrm{Tr}\, X^{p+1}$, [28–30]. This is related to the fact that in the theory with vanishing superpotential, the dimension of the chiral operators $\mathrm{Tr}\, X^n$ can be made arbitrarily small [5–8]. Some of the arguments for the duality of [28–30] apply in three dimensions as well, and it would be interesting to see whether there is a similar duality in this case.

There are of course many other known examples of Seiberg duality in four dimensions, with or without string theory realizations, and it might be interesting to reexamine them in the present context. More generally, Seiberg duality has many applications in field and string theory, some of which might be relevant in three dimensions as well.

Another interesting question of a more general nature is which combination of the $U(1)$ symmetries of an $N = 2$ CS theory is the $U(1)_R$ that enters the superconformal multiplet and determines the scaling dimensions of chiral operators. In four dimensions the answer to this is given by a combination of considerations based on the NSVZ $\beta$-function, $a$-maximization and Seiberg duality [5–8]. In three dimensions, we have Seiberg duality, but the analog of NSVZ and $a$-maximization is not available at present.

Finally, we commented briefly in Section 4 on supersymmetry breaking in $N = 2$ CS theory. It is believed that many such theories have $AdS_4$ gravity duals [31]. It would be interesting to understand the relation between spontaneous supersymmetry breaking in the CS theory and its gravitational dual. This may help develop a holographic understanding of four-dimensional de Sitter vacua of the sort studied in [32].

## Note added

After this work was completed, we received [33], where related issues were considered in the context of fractional M2-brane dynamics.

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